

# ERRATA TO: “MODULE THEORY. ENDOMORPHISM RINGS AND DIRECT SUM DECOMPOSITIONS IN SOME CLASSES OF MODULES”

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Here is a list of errata to my book “Module theory. Endomorphism rings and direct sum decompositions in some classes of modules”, Progress in Mathematics **167**, Birkhäuser Verlag, Basel, 1998.

On Page 5, second line of Section 1.2, delete *non-zero*. This “non-zero” is not a mistake, but is redundant (a module with a proper submodule is non-zero.)

On Page 32, Lines 12–14. The existence of pure-injective envelopes was first published by [Kiełpiński]

Page 72, Notes on Chapter 2. Now I would say that the history of the Krull-Schmidt-Remak Azumaya Theorem began with the paper of [Frobenius and Stickelberger]. In that paper, on Pages 230–231, it was proved that every finite abelian group is the direct sum of a finite numbers of groups of the form  $\mathbb{Z}/p^n\mathbb{Z}$ ,  $p$  a prime,  $n \geq 1$ , in an essentially unique way. This is probably the first example of Krull-Schmidt Theorem.

On Page 92, substitute [Cedó and Rowen] have constructed an example of a local ring  $R$  whose Jacobson radical  $J(R)$  is nil, but  $\mathbf{M}_2(R)$  is not strongly regular with [Cedó and Rowen] have constructed an example of a local ring  $R$  whose Jacobson radical  $J(R)$  is nil, but  $\mathbf{M}_2(R)$  is not strongly  $\pi$ -regular.

On Page 101, last line of the proof of (a)  $\Rightarrow$  (b) in Theorem 4.2, substitute Proposition 2.38 with Corollary 2.44.

On Page 110, delete (3) *If  $A$  is a finitely generated right module of finite Goldie dimension over a strongly  $\pi$ -regular ring  $R$ , then  $\text{End}(A_R)$  is semilocal*, which is false.

On Page 215, last line of the statement of Proposition 9.7, substitute  $k \in J$  with  $k = 1, 2, \dots, n$ .

On Page 223, notice that Puninski’s Theorem 9.19 has been extended from finitely presented modules over a chain ring, that is, finite direct sums of cyclic modules over a chain ring, to arbitrary families of nonzero cyclic right modules over a right chain ring by [Příhoda 06, Proposition 4.2]. (Here there is no mistake in my book, but I wanted to stress this extension due to Příhoda.)

On Page 268, delete the last paragraph, for two reasons. Firstly, Corollary 9.25 has nothing to do with this problem. Secondly, this problem has been completely solved by [Příhoda 04] for serial modules of finite Goldie dimension (Every direct summand of a serial modules of finite Goldie dimension is serial) and by

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[Puninski 01] for arbitrary serial modules (There exist direct summands of serial modules that are not serial).

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#### REFERENCES

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- [Kiełpiński] R. Kiełpiński, *On  $\Gamma$ -pure injective modules*, Bull. Acad. Polon. Sci. Sr. Sci. Math. Astronom. Phys. **15** (1967), 127–131.
- [Příhoda 04] P. Příhoda, *Weak Krull-Schmidt theorem and direct sum decompositions of serial modules of finite Goldie dimension*, J. Algebra **281** (2004), 332–341.
- [Příhoda 06] P. Příhoda, *On uniserial modules that are not quasi-small*, J. Algebra **299** (2006), 329–343.
- [Puninski 01] G. Puninski, *Some model theory over an exceptional uniserial ring and decompositions of serial modules*, J. London Math. Soc. **64** (2) (2001), 311–326.

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